10 The Mapping of Family Composition Preferences: a scaling analysis

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1. INTRODUCTION

As a variable, "family size and composition" has the sociologically unusual property of being clearly related to the defining "objective," numerical, characteristics of the stimuli, namely sex, and number of children. It is perhaps because these characteristics are so unambiguously defined that interest has centered in the main on issues other than simply identifying the factors which enter the preference judgments. Two questions have been especially salient.

1. Are subjective differences in family size the same as the objective differences? If not, are they systematically related?
2. How do subjects combine the defining characteristics in order to arrive at their preference scales?

The first problem, a familiar enough one in psychophysical contexts, has been discussed in detail by Goldberg and Coombs (1963) in their analysis of judgments of the difference made by families of varying size and composition to a mother's "satisfaction and problems." As in other studies, one main conclusion was that the subjective distance between adjacent numbers of children systematically decreases with the increasing number of children. In particular, the first prime interval (the subjective difference between having no children and having one child) was found to be very much larger than subsequent ones. If subjective scales depart systematically and markedly from the objective (equal interval) scale, then it is presumably the subjective scale which should feature in any theory explaining subjects' preferences.

In a recent paper, Coombs and his colleagues (1973) present a number of alternative models of how subjects combine information to arrive at overall preference rankings for families of different composition and size. They go on to test whether the subjects' data possess a number of ordinal properties which axiomatic measurement theory shows to be necessary conditions for obtaining an additive numerical representation. Two models which he presents are of particular relevance to the analysis presented here:

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Model 1.

$$R(i \text{ and } j) = M_k(u(s_i) + u(d_j)).$$

That is, the rank assigned by a given individual k to the family composition stimulus of i sons and j daughters is a monotonic (decreasing) function of the utility assigned to i sons added to the utility assigned to j daughters (The utilities are assumed to be single-peaked, having a single maximum at the subject's 'ideal point' over the variable concerned).

Model 2.

$$R(i \text{ and } j) = M_k(u(x_{ij}) + u(y_{ij}))$$

where

$$x_{ij} = s_i + d_j,$$

$$y_{ij} = s_i - d_j.$$

That is, the preference rank assigned by a given individual to the composition (i and j) is a monotonic function of the addition of two (new) variables: x_{ij} (which is the total number of children) and y_{ij} (which represents the preponderance of sons over daughters).

Both of these models are straightforward and appealing. They postulate that preference for a particular composition of stimuli is determined by the additive effects of the attributes making up the composition; they differ only in the nature of the attributes. The crucial assumptions are that only two characteristics systematically enter the preference judgment and that the utilities combine additively. If they do *not* combine additively, but interact in producing their effects, this may be due either to the nature of the scales (i.e., a transformation of the values may render the preferences additive) or to the inherently nonadditive nature of the process of composition. Axiomatic measurement theory (Krantz *et al.,* 1971, Chap. 6) provides a series of tests for whether (a possibly ordinally rescaled version of) the data are compatible with an additive model, and several scaling procedures exist for estimating the resulting utility values.

In this paper, a less stringent and more general approach is adopted, oriented more to exploring the characteristics of the data than to the confirmation of particular hypotheses about them. But by tolerating greater error, and confronting the data with models of considerable generality and variety it will be possible to investigate the tenability of certain assumptions which Coombs' models make.

The strategy followed here will be to analyze a set of preferences for families of different sizes and composition within the framework of what might be termed the "general distance model," analogous to the general linear model.

The various stimuli (family compositions) are viewed as points located in a space of unknown dimensionality. As will be seen, previous investigators have postulated a dimensionality as high as six; hopefully no more than the two defining dimensions (number of sons and number of daughters)—or some simple function of them—will be necessary to accommodate the data. The first part of the paper will be concerned with identifying these dimensions in terms of two rather different conceptualisations (models) of how preferences are to be interpreted. The second part of the paper will concentrate on the composition problem: given a representation (or configuration) of the stimuli, in what ways is this information combined in arriving at overall preference rankings? The aim will be to compare the rationale of several approaches towards explaining such preferences, and examine the cognitive and substantive implications of the solutions obtained by applying the models to the data.

Two Spatial Models

The spatial models considered here share a number of assumptions. First, it is useful when conceptualising preference to think of subjects as having a "cognitive map" or "internal representation" of a set of stimuli—in this case, of families of different size and composition. For many other sorts of cultural objects, it is hazardous to assume that subjects perceive the stimuli the same way (or share the same cognitive map) and models have to be developed which can accommodate highly dissimilar cognitions.[2] However, differences in preferences for families of differing size and sex composition are unlikely to spring primarily from cognitive differences; differences in *evaluation* are much more likely to be the major source of variation.

Within the context of spatial models, two main ways have been suggested for representing preference. In the "locational" (distance or unfolding) model, each subject is assumed to have a unique point of preference which is defined by the coordinates of his maximum preference on each dimension of the map, and it is assumed that his preference (or "utility" function over the map) is single-peaked"[3] or has only one maximum. The subject's rank order of preference for the stimuli is interpreted as giving information on the rank-order of distances between the location of his "ideal point" and the location of the stimuli. Preference for a set of objects is thus viewed as a decreasing function of their distance from the subject's ideal point in the same cognitive space.[4] An alternative linear, or vector, model (Tucker and Messick, 1963; Carroll, 1964) also assumes that a set of subjects share a common cognitive map, but their preferences are now represented as a direction or vector in that space. The subject's order of preference is interpreted as giving information on the rank order of the projections of the stimuli on his vector.

These two models differ principally in what assumptions they make

[2]Carroll and Chang (1970) have developed a particularly powerful model (INDSCAL) for representing individual differences in *perception* by assuming that each subject attaches differential (possibly zero) importance or weight to a common set of dimensions. In this case a similar pattern of weights represents a specific point of view.

[3]A concept introduced by Black (1948), and developed in this context by Coombs (1964), Arrow (1951), Luce and Raiffa (1957), and Carroll (1972).

[4]This assumption is substantiated in some studies, where subjects' preferences are inferred from information on their judgments of the similarities between stimuli (see Klahr, 1969; and Steinheiser, 1970).

TABLE 1

Summary Information on Rank Scores for Different Family Size Compositions[a]

Stimuli	Code	Range	Mean	Variance	Skewness	Kurtosis
0 children		0-4	0.2	0.4	3.8	15.8
1 son	1S0D	0-12	3.6	7.3	1.1	0.3
2 sons	2S0D	2-17	8.1	13.1	0.6	–0.3
3 sons	3S0D	5-16	9.7	6.6	0.3	–0.4
4 sons	4S0D	4-18	10.0	9.6	0.3	0
5 sons	5S0D	0-20	8.7	20.6	0.2	–0.4
1 daughter	0S1D	1-11	2.6	4.4	1.7	3.2
2 daughters	0S2D	2-13	5.6	7.7	0.9	0.1
3 daughters	0S3D	2-12	6.2	5.0	0.5	–0.4
4 daughters	0S4D	1-13	5.8	5.9	0.3	0
5 daughters	0S5D	0-13	4.8	11.8	0.6	–0.6
1 son, 1 daughter	1S1D	4-19	11.0	17.5	0.4	–1.0
2 sons, 1 daughter	2S1D	10-20	15.1	6.1	0.1	–0.3
3 sons, 1 daughter	3S1D	10-19	15.9	3.5	–0.6	0.2
4 sons, 1 daughter	4S1D	5-20	14.8	12.6	–1.0	0.7
1 son, 2 daughters	1S2D	6-20	12.5	7.5	0.2	0.1
2 sons, 2 daughters	2S2D	13-20	17.9	2.3	–0.8	0.8
3 sons, 2 daughters	3S2D	12-20	18.4	4.6	–1.6	1.8
1 son, 3 daughters	1S3D	2-18	12.0	8.7	–0.6	0.8
2 sons, 3 daughters	2S3D	9-20	17.1	5.6	–1.2	1.0
1 son, 4 daughters	1S4D	1-18	10.1	17.5	–0.3	–0.8

[a]Highest preference is a rank of 20, and lowest preference has a rank of 0.

about how the subject combines information relating to the dimensions of the space when arriving at his overall preference ordering. The vector model makes the simplest assumption, implying that subjects simply collapse the space by projecting the stimuli on to *one* dimension oriented towards the most highly preferred region of the space. The angle between a subject's preference vector and a dimension spanning the space can then be interpreted as measuring the importance of the contribution which that dimension makes to his preference judgment. It also implies, rather less acceptably, that the subject's preference increases unboundedly along each dimension. In the present example, if one dimension is the number of children, this means that if I highly evaluate a family size of x, then I will value a family size of $x + 1$ even more highly. By contrast, the distance model implies (because of the single-peakedness of the preference function) that I will have a unique point of maximum preference on each dimension, and that my preference will decrease systematically (though not necessarily symmetrically nor linearly) in *every* direction, which is a far more reasonable assumption. On the other hand, the distance model assumes that the subject employs a much more complex rule for combining information—namely that absolute differences on each dimension, which are transformed by the same power-function, combine additively to produce the overall preference value.[5]

[5]See Beals *et al.* (1968, pp. 133-5). This is simply a verbalisation of the general distance function.

2. INTERNAL SCALING ANALYSIS

In an extended examination of the applicability of scaling models to the analysis of preference data, Delbeke (1968), presents rank orderings of preference for the 21 family-composition stimuli given in Table 1. The data were obtained in the form of complete pair comparisons,[6] from a group of 80 psychology students at the University of Louvain, matched by sex and by socioeconomic variables.

Table 1 presents summary statistics on the preference "vote-count" data.[7] Perhaps the two most striking inferences to be drawn from these data are the almost universal dislike for childless families, and the high preference given to large family sizes, with a composition of 3 sons and 2 daughters having highest aggregate preference. In general, the following propositions seem to hold, and are broadly supported by other empirical research studies (Coombs *et al.,* 1973; Westoff *et al.,* 1961; 136 *et seq.*; Freedman *et al.,* 1960; Ryder and Westoff 1965):

(i) For each given *single sex* family size, all-boy families are preferred to all-girl families. Moreover, the difference in average preference (or marginal utility) of boys increases systematically with the overall size of family (at least up to 4 sons).
(ii) For any family size, a mixed sex-composition is preferred.
(iii) A preponderance of boys is preferred in mixed-composition families.

The distributions of rank preferences, "no children" (and "1 daughter") are far from normal, being highly skew and peaked, with very little variance; moreover, some distributions are markedly bimodal. Because of the very small variance of the stimulus "no children," this has been removed from subsequent scaling analyses.[8]

In his analysis of these data, Delbeke used both a vector model and a distance model,[9] and he accepted a solution of four orthogonal dimensions from both the vector model and from the distance model. After

[6] I am very grateful to Dr. Delbeke who kindly provided me with a copy of the original pair-comparison data.

[7] The basic data exist in two forms—80 dominance (0,1) matrices of pair comparison judgments, and as 80 rankings or "preference votes" obtained by summing across rows of each pair comparison matrix. Delbeke (pp. 123-4) reports the data as preference rankings.

[8] Its inclusion in the multidimensional scaling analysis very badly distorts otherwise interpretable configurations. In the vector model, nearly universal rejection has the effect of locating that stimulus point in the opposite direction to the vector of the average subject, and in the distance model it has the effect of locating it well beyond the next least preferred stimulus point. In the case of the distance model, a universally rejected stimulus must be located at a point maximally distant from most subjects; its precise location can be very unstable, but it tends to be located on a circle (or hypersphere) at fixed distance from the centroid of the subject-point locations.

[9] The vector model was a variant of Tucker's scalar products model (see above) and the distance model was Gleason's (1967) method for nonmetric multidimensional unfolding analysis.

rotation he identifies these dimensions (Ryder and Westoff, 1965, pp. 75-77, 102-103). (a) Mixed sex vs one sex composition; (b) Number of children; (c) Number of sons; (d) Number of daughters.[10]

In subsequent reviews of this work, Carroll (1970) and the present author (Coxon, 1969) were of the opinion that the high number of dimensions (at least compared to the defining characteristics of the stimuli) could well be due, at least in part, to the deficiencies of the then-available models and computer programs. More recently developed, more robust procedures yield a more acceptable solution to these data, and lead to a better explanation of them.

Vector Model Analysis

Carroll (1964) presents a vector model for preference data (MDPREF) in which the location of the stimulus points and the directions of the subject vectors are obtained simultaneously, for a given dimensionality. Given a user-specified dimensionality (r), the preference matrix, **S** (of n individuals by m stimuli) is factored into the product matrix

$$\mathbf{S} = \mathbf{X}\mathbf{Y}^T$$

where **X** gives the direction cosines of the unit-length vectors from the origin of the space for the n subjects, and **Y** gives the coordinates of the locations of the m stimuli points. After defining an index of agreement between data and a given configuration, Carroll (1964, 2) shows that it is maximised by an Eckart-Young decomposition, producing **X** and **Y** matrices which give the best least-squares estimate (for a given dimensionality) of the preference matrix, **S**.

This procedure was applied to both the preference scores and the pair-comparison data. Solutions were sought in two and three dimensions, for men and women subjects separately. The preference score and pair comparisons data produce almost identical solutions in each case.[11] Inspection of the roots of the first-score matrices strongly suggests that a two-dimensional solution is adequate and strongly counterindicate the four- initiality six-dimensional solution accepted by Delbeke.[12] The two-dimensional solutions for males and females are presented in Fig. 1.

It is fairly easy to interpret these configurations: mixed/unmixed sex

[10]In the vector model, these four dimensions account for 91% of total estimated communality.

[11]The average number of intransitive (circular) triads in the subjects' preference judgments is 21.4 (median at 17) out of a possible 385, and the standard deviation is 19.7. By contrast, the *expected* number of circular triads under the assumption of random choice is 333, with a standard deviation of 15.8 (Kendall, 1962, p. 156), giving an average coefficient of consistence of .94. No subject's triads come even within 20 standard deviations of chance expectation. Future users of this data set should note that the number of intransitivities given by Delbeke for subjects 5, 27, 59, and 71 are incorrect, and should be 1, 30, 5, 17, respectively.

[12]The percentage of variation accounted for in the present analysis is (by dimension) 75, 14, 6, 1 (males) and 75, 11, 6, 2 (females). Carroll (1970, p. 279) discussed in detail the reasons why Delbeke accepted such a high dimensional solution. In part, this is due to the mistaken decision to factor the double-centered matrix of preferences.

composition forms the first dimension, and overall family-size forms the second. But both factors are systematically distorted, mirroring the spread of the individual preference-vectors on the unit-circle. A "number of sons vs number of daughters" factor is certainly identifiable in the residual third dimension[13] but the amount of variation it explains is very small.

While there is a good deal of individual variation (represented, for instance, by the angle between the extreme subject vectors) all subject-vectors

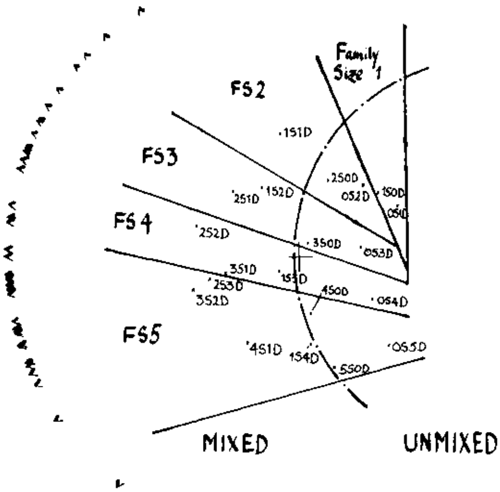

Fig. 1A. Family composition preferences: Male subjects. Internal analysis, vector model (MDPREF) 2-dimensional solution. Endpoints of subjects' vectors denoted by arrowheads.

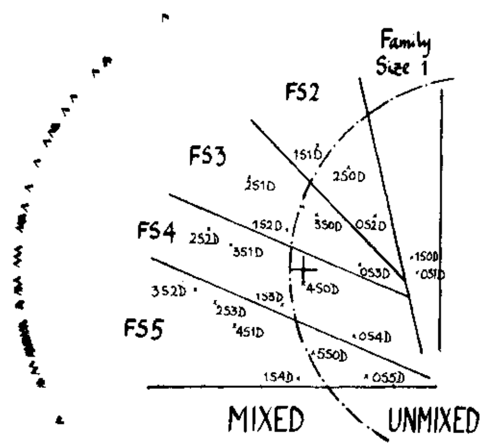

Fig. 1B. Family composition preferences: Female subjects. Internal Analysis, vector model (MDPREF); 2-dimensional solution.

[13]The stimuli with the highest positive coordinates on Dimension III are 5*S*, 4*S*, 3*S*, 2*S* and those with the highest negative coordinates include 1*S*4*D*, 1*S*3*D*, 5*D* and 4*D*.

are oriented positively towards mixed families, and the main source of individual variation occurs on the *size* of family. Males generally prefer larger families, and there is a somewhat larger number of vectors directed to smaller family size among females.

Despite what will turn out to be the inadequacies of the vector model representation, the inferences made above on p. 4 can be recognised in this representation in the following ways.

(i) *All-boy families are preferred to all-girl families, for given single-sex family-sizes.* Concentrating on single-sex pairs of fixed family size (such as (1S0D, 0S1D), (2S0D, 0S2D), . . ., (5S0D, 0S5D)), the points representing all-boy families are systematically to the left of (more highly preferred to) all-girl families of the same size.

(ii) *A mixed sex-composition is preferred within a given family-size.* Within each given family-size "strip," the points representing mixed families are consistently to the left of (preferred to) those representing unmixed family composition.

(iii) However, the inference that *within* a given type of mixed-composition family, a preponderance of boys is preferred does not seem to hold.

Distance Model Analysis

Coombs' (1964) unfolding analysis of preference data provides a very appropriate model for data of this sort. In this distance model, both stimuli and individuals are represented as points in a (possibly multidimensional) space.

Each subject is defined by his point of maximum preference in the stimulus space, and his preference is interpreted as a monotonic function of the separation between his "ideal point" and the location of the stimuli.

Although some progress was made in earlier years towards developing an algorithm for analyzing fallible data according to the multidimensional unfolding model, no satisfactory solution existed before 1966, when nonmetric multidimensional scaling procedures were adapted to "rectangular" data-matrices (i.e., defined by two *distinct* sets of entities). Since comparability is only assumed between rows of the matrix in such a procedure,[14] relatively few constraints exist for obtaining a solution, and multidimensional unfolding procedures are still especially prone to degeneracy and nonoptimal solutions. Delbeke used Gleason's (1967) algorithm (Delbeke, 1968, pp. 101-5). The data were reanalyzed by Roskam's[15] (1968) procedure which is probably the most robust presently available. This was applied to the preference-score data, and solutions were sought in 3 and 2 dimensions.

The solutions for male and female were once again virtually identical, and therefore analysis was run on the full 80 cases. The two-dimensional

[14]For example, the person's ordering is assumed to be in the same metric, but orderings from different individuals are not considered comparable.

[15]See Roskam (1969, II, pp. 18-21) and Lingoes and Roskam (1971).

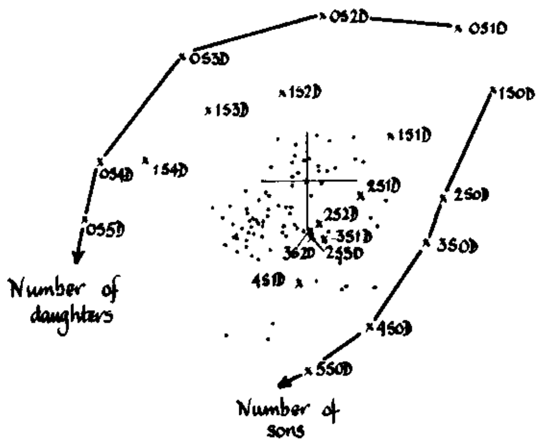

Fig. 2. Family composition preferences: Male and female subjects. Internal analysis, distance model (unfolding). 2-dimensional solution.

solution is presented in Fig. 2.[16]

The most striking feature of the 2-dimensional distance representation (unlike the vector representation) is the obvious salience of the defining characteristics of the stimuli—the number of sons, and the number of daughters.[17]

In terms of their definition, the stimuli are arranged in the form of a regular semilattice like that presented in Fig. 3a. These "dimensions" are clearly discernible, in a somewhat distorted and correlated form, in the unfolding analysis solution in Fig. 2; they are by no means so evident in Delbeke's solution (p. 164). Compared to the defining, "rational" configuration, the one produced by the scaling analysis is systematically distorted:

(i) There seems to be no consistent pattern in the size of the prime intervals along the "number of sons" (S) and "number of daughters" (D) dimensions. In particular, the size of subjective family-size differences do not systematically decrease; in D, the intervals are fairly equal (except for the last) and in S there are no consistent differences in size of interval.

(ii) While the S "dimension" is fairly linear, the D "dimension" is markedly curvilinear, although it is approximately linear in the very small values.

[16]The badness-of-fit measure for such data (stress$_2$ based on Kruskal's fitting quantities) is .069 for 3 dimensions, and .124 (for 2 dimensions). On several criteria, the two-dimensional fit is acceptably low, and certainly gives a more interpretable result than that for 3 dimensions.

[17]The two "objective" properties of "number of sons" and "number of daughters" were separately fitted as vectors in the stimulus space of the distance model according to Chang and Carroll's (1970) procedure, involving maximising the linear correlation between stimulus coordinates and external property values for these stimuli. The "number of sons" fitted vector correlated very highly (.926) with stimulus coordinates, as did the "number of daughters" (.928).

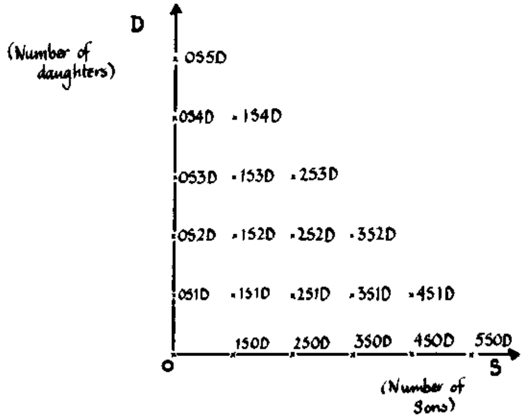

Fig. 3A. Family composition: Rational configuration of stimuli in terms of number of sons and number of daughters.

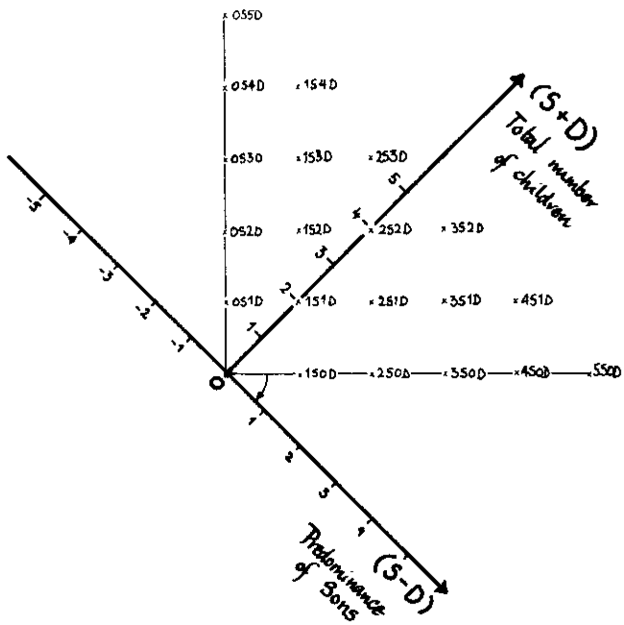

Fig. 3B. Family composition. Rational configuration of stimuli in terms of Coombs' number of children and preponderance of sons (45° rotation of number of sons and number of daughters).

(iii) Despite the dependence between *S* and *D*, and the nonlinearity of *D*, the relative positioning of the smaller, mixed family-composition, stimuli is not badly distorted. But the larger family-sizes which are equally mixed in terms of sex composition, virtually collapse on to the same point.

Do these "distortions" represent genuinely psychological processes, or are they methodological artefacts? Unfortunately, the latter cannot be ruled out.

Models which fit both stimulus points and subjects points (or vectors) *simultaneously* are particularly sensitive to the distribution of subjects' judgments, and in order to fix stimulus points reliably, it is necessary to ensure an adequate number of cases and spread of ideal points or vectors. But mixed-composition, large family-size stimuli are very extensively preferred in these data, as we have seen. The multidimensional unfolding solution must consequently locate these stimuli close to the center of individual ideal points, giving rise to the "collapsing distortion" noted in (iii) above. Whenever uneven, heavy, concentrations of individual preferences occur, major distortions are likely in fixing the location of stimulus points. For this reason it would be hazardous to read too much into the solution.

Nonetheless the simplicity of the distance representation is appealing, not least because it allows the separate "mixture" dimension so evident in the vector representation to be dispensed with and interpreted as a straightforward combination of the "number of sons" and "number of daughters" dimensions of the distance-representation.

3. EXTERNAL ANALYSIS OF THE DATA

The unfolding model of preference implicitly assumes that subjects agree in their cognitions of the objects being judged, and that the main source of individual variability is the different *evaluations* which subjects give to the stimuli. But, if subjects differ in their perceptions of the stimuli as well as in their evaluations of them, then an unfolding analysis will not be able to recover the cognitive space correctly, and it will not be possible to decide whether the reason for the badness of fit is due to differential perception, or to the fact that the distance model is not appropriate.[18] To mirror this fact, Carroll (1972, p. 114) proposed a distinction between models for the *internal* analysis of preference data such as Coombsian unfolding (where both stimuli *and* subjects are parameterized from the same set of preference data) and *external* analysis (where the preference data are fitted in an independently derived space). In many cases of preference analysis, an independent set of judgments of similarity between the stimuli is made by the same subjects, and can be scaled to obtain such an a priori cognitive space.[19]

However, separate similarities data are not available in this study. But

[18]If considerable differences in perception do exist, then the results from an unfolding analysis may lead, quite incorrectly, to abandoning a distance model. It might be that all subjects' preferences are in fact a monotone function of the separation of their ideal points and the stimuli points, but that the distances refer to different cognitive spaces.

[19]If considerable, systematic, individual differences in perception are then found to exist, further analysis can be done within each group of subjects having relatively homogeneous perceptions.

there is no reason why the a priori space must be derived from similarities—any external configuration which can be related meaningfully to the preference data may be used. In this case, since the stimuli are defined in terms of the composition of two "objective" or "physical" variables, and we know that a two-dimensional representation does not seriously distort the subjects' judgments, the semilattice-structure presented in Fig. 3 can act as the a priori space.

How do the subjective preferences relate to the "objective" characteristics of the stimuli represented by the semilattice?[20] Carroll (1972) developed a hierarchy of four external models for "preference mapping" (collectively referred to as PREFMAP) which both generalises and particularises Coombs' unfolding distance model for mapping preference data:

		Model allows:		
Phase or level of PREFMAP	Name of model	Differential rotation of axes	Differential weighting of axes	Differential location of ideal points
I	General unfolding	+	+	+
II	Weighted unfolding	–	+	+
III	Simple unfolding (Coombs)	–	–	+
IV	Vector	–	–	–

Formulated in most general terms, the preference scale values of individual i for stimulus j (s_{ij}) are assumed to be a function of the (squared) distance between his ideal point y_i and the stimuli locations x_j:

$$s_{ij} = F(d_{ij}^2) = a_i + bd_{ij}^2 + e_{ij} \; (b \geqslant 0).$$

The four models or levels of the PREFMAP hierarchy are distinguished in terms of how the squared distances (d_{ij}^2) are defined. In Level I subjects are permitted (i) to rotate the reference dimensions of the space and, (ii) *then* differentially weight them. As Carroll describes it:

> We allow distinct individuals additional freedom in choosing a set of 'reference axes' . . . and then to weight differentially the dimensions defined by this rotated reference frame, in addition to being permitted an idiosyncratic ideal point. (Carroll, 1972, p. 120).

A subject is assumed to apply his *own* orthogonal rotation T_i to the axes in which both the stimuli and ideal points are located, and then weight the rotated dimensions. If x_{ja}^* represents such transformed stimulus coordinates, y_{ia}^* the transformed ideal point coordinates, and w_{ia} represents the *evaluative weight* applied to dimension a then:

$$s_{ij} = F_i \, (d_{ij}^2)$$

[20]Strictly, it should be possible to perform a two-stage analysis, first enquiring how subjects' *similarities* data relate to the physical attributes and secondly enquiring how the preferences relate to the cognitions (the usual external analysis).

where

$$d_{ij}^2 = \Sigma_a w_{ia} (y_{ia}^* - x_{ja}^*)^2,$$

i.e., a Euclidean distance in an individually-rotated and weighted "private space."

In Level II individual rotations are excluded, but a subject is assumed simply to apply an evaluative weight w_{ia} to each dimension, so that

$$s_{ij} = F_i (d_{ij}^2)$$

where now,

$$d_{ij}^2 = \Sigma_a w_{ia} (y_{ia} - x_{ja})^2,$$

i.e., a Euclidean distance in a weighted or differentially scaled "private space."

In Level III—a simple Coombsian Unfolding case—subjects are each represented by an ideal (most preferred) point in the cognitive space.[21] The closer a stimulus is to an individual's ideal point, the more he will prefer it. Since differential weighting is excluded, a given difference on a particular dimension is assumed in this model to have the same meaning and contribute identically to the overall distance for every subject, so that

$$s_{ij} = F_i (d_{ij}^2)$$

where now,

$$d_{ij}^2 = \Sigma_a (y_{ia} - x_{ja})^2,$$

i.e., the normal Euclidean distance metric.

In Level IV, the preferences are assumed to be a simple linear function of the stimuli values themselves:

$$s_{ij} = a_i + \Sigma_a b_{ia} x_{ja}$$

and this becomes the *external* analogue to the vector model encountered earlier.[22]

Since in the metric version of the PREFMAP models a subject's preferences are assumed to be linearly[23] related to the (weighted, transformed or simple) distances between the stimuli locations and his ideal point,

[21]In fact, the PREFMAP version of unfolding analysis permits subjects to be represented as having negative ("anti-ideal") points on one or more dimensions. Where this occurs, the subject's preference function is interpreted as U-shaped, containing a single minimum, indicating his point of least preference on the dimension concerned. No negative values occurred in this analysis.

[22]Carroll (1972) has proved that the vector model is a special case of the simple distance (unfolding) model. As an ideal-point of the distance model is moved further and further out from the origin of the space, a circular isopreference contours (joining points of equal preference for this subject) more and more closely approximate straight lines in the vicinity of the stimuli points; and isopreference "contours" in the vector model consist of precisely such straight lines, perpendicular to the subject's vector.

[23]In the program implementing the PREFMAP models an option also exists for the nonmetric (i.e., monotonic) regression of preference values on the model distances (or projections on the subject vector in the case of model IV).

product-moment correlations between a subject's preference values and those estimated in terms of a particular model can be calculated to provide a useful measure of individual goodness of fit. Moreover, since each model is a special case of the higher one in the hierarchy, it is possible to use variance analysis to test whether the more general model explains a significantly greater amount of variation than the more particular one.

It seemed reasonable to assume that the source of greatest individual variation in family size preferences would be the sex of the subject. Hence subjects were again divided into male and female groups, and analyzed separately.

The data were analyzed in terms of Carroll and Chang's hierarchy of four preference-mapping models (PREFMAP). Both metric and nonmetric fitting functions were employed.[24] The overall goodness of fit measures of the models for the "average subject" of each group are presented in Table 2.

Two things are particularly notable about the PREFMAP analysis—the high overall correlations, and the very marked superiority of the distance models over the vector model for explaining these data, in terms of the rational configuration. Moreover, the increase in goodness-of-fit from the simple (III) to the weighted (II) distance model is so marginal that the simple distance model can be taken unequivocally to be the appropriate model; differentially weighted dimensions are not necessary for explaining these subjects' preferences. To summarise:

(1) The overall (root mean square) correlations show excellent fit of the data to all the distance models (i.e., except for the vector Model IV). This holds for both male and female groups, and for the metric and nonmetric versions of the models. Moreover, the individual correlations (not presented here) are uniformly and consistently high, except in the case of the vector model, where considerable differences are evident.

(2) Among the distance models, the simple distance (unfolding) model has only marginally lower correlations than the weighted and general distance models. There is no need to invoke differential (idiosyncratic) weighting or rotation of axes in order to account for subjects' preferences.

(3) In the vast majority of cases, the goodness-of-fit (F-ratio) values indicate that Model III (simple distance) fits individual subject's preference data very significantly better[25] than Model IV (vector). With the possible exception of one case, every subject should be assigned to the simple distance model.

[24]Options used included normalising preference values to unity in all analyses, and, in the nonmetric analyses, using a difference criterion of .001 and primary approach to ties in the block-monotone fitting function.

[25]All the individual F-values very considerably exceed the critical ratio of 8.53 (for $df1 = 1$, $df2 = 16$, significance level of .01) except for 1 male ($p > .10$) and 2 females ($.01 < p < .05$).

TABLE 2

Goodness of Fit between Data and PREFMAP Models

(a) Correlations (average subject)

	Metric						Nonmetric					
	Male			Female			Male			Female		
Model	*rms*[a]	Min	Max	*rms*[a]	Min	Max	*rms*[a]	Min	Max	*rms*[a]	Min	Max
I	.9368	.9141	.9934	.9478	.7994	.9922	.9904	.9178	.9993	.9868	.9350	.9995
II	.9446	.7852	.9933	.9447	.7932	.9907	.9904	.8832	.9995	.9858	.9049	.9998
III	.9415	.7832	.9932	.9356	.7919	.9904	.9885	.8796	.9995	.9812	.9056	.9995
IV	.6135	.1530	.9844	.7004	.1440	.9080	.7821	.4085	.9978	.8389	.5933	.9915

[a]Root mean square.

(b) Analysis of variance between models

	Male		Female		Male		Female	
Model (*df*)	*F*-ratio	Significance	*F*-ratio	Significance	*F*-ratio	Significance	*F*-ratio	Significance
I,II(1,14)	0.0005	*ns*	0.0014	*ns*	1.7375	*ns*	–.1876	*ns*
II,III(1,15)	0.0019	*ns*	0.0008	*ns*	2.6602	*ns*	5.9793	*ns*
III,IV(1,16)	195.0966	<.001	202.5904	<.001	1737.4939	<.001	563.1711	<.001

The simple distance solution is presented in Fig. 4, for the metric case. The main point of interest in the external analysis is no longer the relative positioning of the stimulus points, but the location of the subject points. First, there is little distinction to be made between the male and female distributions of ideal points—if anything, females tend on average to prefer

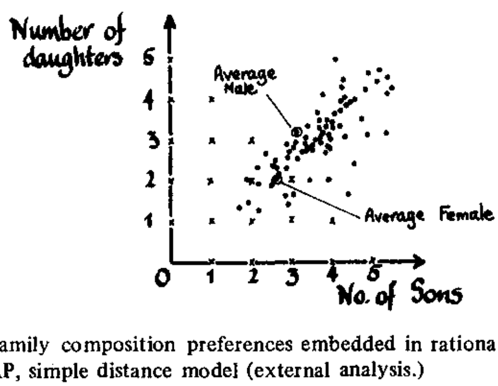

Fig. 4. Family composition preferences embedded in rational configuration according to PREFMAP, simple distance model (external analysis.)

somewhat larger family sizes than males, and there is a slight tendency for them to prefer boys to girls. More striking than the differences are the similarities. Once more, the location of the distribution of ideal points emphasizes the salience of equally-mixed sex-composition in the preferences of these subjects.

In view of these results, little more can be said about the "multidimensional psychophysics" question—the data fit the simple unfolding model so well that the conclusion must be that, at least for these subjects, there is little need to invoke systematic individual differences in cognition of family-size and sex composition. Moreover, the preferences can be quite adequately interpreted in a directly Coombsian manner, as a simple function of the proximity between the subject's most preferred location on the two constituent "physical" dimensions (defining his ideal point) and the locations of the stimuli.

4. CONCLUSION

The reason why the distance model gives a much better fit to the data than the vector model is probably related to the assumptions which each model makes about how information is combined into overall preferences. The distance function in this case provides a more acceptable subjective composition-rule than the vector model for combining the number of sons and number of daughters information—namely that *differences* on each dimension combine *additively* to produce the similarity (preference) judgment.

Shepard (1964, p. 270) argues that compressing multidimensional information into an overall decision consists of two distinct problems—the specification of the rules of combination, and the problem of assigning appropriate weights to the component factors. The second problem does not seem to arise here—from the results of Model II, equal weighting of the two dimensions seems to hold for each subject. However, the fact that a *Euclidean* distance model produced such excellent goodness of fit is hard to explain. Attneave (1950), Torgerson (1952), Shepard (1964) and Hyman and Well (1967) argue that where dimensions of judgment are particularly salient or culturally obvious (and are few) the much simpler "city block" distance metric (which simply asserts that absolute differences on dimensions combine in an additive way to produce the dissimilarity judgment) provides a better fit, and is more appropriate. A priori this argument would seem to apply to these data—after all, few sociological data have such clear and perceptually distinct dimensions. But as PREFMAP is currently programmed, the hypothesis of the greater applicability of the city-block metric could not be tested.

There is one interpretation which suggests that the Euclidean metric may, after all, be appropriate. Hake and Rodwan (1966, cited in Hyman and Well, p. 347) and others have suggested that the property of rotational invariance peculiar to Euclidean space is "a highly adaptive property for an organism that seeks invariance and stability in its perceptual world," and Hyman and Well (p. 247) argue that "the more two component dimensions interact, the more the appropriate spatial model will deviate from the city block metric." In slightly reworded form, "interaction" referred to by Hyman and Well, and by the Coombs, and evident in the present analysis, is the "mixture" phenomenon already noted. Two pieces of information support this interpretation:

(i) Dimensional interaction was evident in the fact that the "number of sons" and "number of daughters" vectors are nonorthogonal in the *internal* analysis of these data.

(ii) When the data were analyzed by level I of the PREFMAP hierarchy (the General Unfolding Model) the vast majority of individual rotations took the form of a clockwise rotation[26] of the rational configuration through between 40° and 50°, and the optimal rotation is in the vicinity of 45°. However, if the dimensions defining the "rational configuration" in Figure 3a are rotated in this way, it becomes clear that the new dimensions correspond respectively to *number of children* ($S + D$), and *predominance of sons* ($S - D$). (This is illustrated in Fig. 3b). These rotated dimensions correspond precisely to the two variables which enter the Coombs' Model II, and which, they argue, provide a theory which is substantively and methodologically superior to one based upon the number of sons and number of daughters. It would seem that empirical analysis supports measurement-theoretic analysis in this instance.

This leaves unresolved the theoretical question of what composition-function provides the best explanation of the preferences. The Coombs' model of simple additive composition is, of course, a good deal simpler than that implied by the distance model, although their Model II can be interpreted as a special case of one particular distance model (namely, a city-block metric distance within the rotated dimensions). But the theoretical question cannot be settled by a scaling analysis.

These considerations highlight what is probably the most significant point of this analysis–recently developed models in multidimensional scaling and related areas allow for more sophisticated analysis than our data can usually support. In particular, the substantive presuppositions of the models are surprisingly strong and testable, and yet they cannot be tested without independent evidence. Decision-making about family size lends itself well to systematic experimental research and theoretical development. Moreover, the study of how relevant information is in fact combined will yield rich comparative data for other, more diffuse, areas of social cognition and evaluation.

[26]The (clockwise) angle of rotation of the configuration presented in Fig. 3a for the "average subject" in Level I is, for males: 68° (metric) and 54° (nonmetric); for females: 42° (metric) and 43° (nonmetric).

REFERENCES

Arrow, K. J. (1951), Social Choice and Individual Values. Wiley, New York.

Attneave, F. (1950), "Dimensions of similarity," American Journal of Psychology **63**, 516-556.

Beals, R., D. H. Krantz, and A. Tversky (1968), "Foundations of multidimensional scaling," Psychological Review **75**, 127-142.

Black, D. (1948), "On the rationale of group decision making," J. Political Economics **56**, 25-34.

Carroll, J. D. (1964), "Nonparametric multidimensional analysis of paired comparisons data," Bell Laboratories mimeo, Murray Hill, (1970); "Review of Delbeke (1968)," Psychometrika **35**, 178-281, (1972); "Individual differences and multidimensional scaling," *in* Shepard *et al.*, Multidimensional Scaling; Theory and Applications in The Behavioral Sciences, Vol. 1. Seminar Press, New York.

Carroll, J. D. and J. J. Chang (1970), "Analysis of individual differences in multidimensional scaling via a *N*-way generalization of "Eckart-Young" decomposition," Psychometrika **35**, 283-319.

Chang, J. J. and J. D. Carroll (1970), "How to Use PRO-FIT, a computer program for property fitting," Bell Laboratories, mimeo, Murray Hill, N.J.

Coombs, C. H. (1964), A Theory of Data, Wiley, New York.

Coombs, C. H., G. H. McClelland, and L. C. Coombs (1973). "The measurement and analysis of family composition preferences." University of Michigan, MMPP, mimeo. Ann Arbor.

Coxon, A. P. M. (1969), "Review of Delbeke (1968)," Sociological Review **17**, 403-406.

Delbeke, L. (1968), Construction of Preference Spaces. Publications of the University of Louvain, Louvain.

Freedman, D. S., R. Freedman, and P. Whelpton (1960). "Size of family and preference for children of each sex." American Journal of Sociology **66**, 141-146.

Gleason, T. C. (1967), "A general model for nonmetric multidimensional scaling," University of Michigan, MMPP, mimeo, Ann Arbor.

Goldberg, D., and C. H. Coombs (1964), "Some applications of unfolding theory to fertility analysis" *in* Emerging Techniques in Population Research, (Proc. 1962 annual conference of Milbank Memorial Fund), New York.

Hyman, R. and A. Well (1967), "Judgments of similarity and spatial models," Perception and Psychophysics **67**, 233-248.

Kendall, M. G. (1962). Rank Correlation Methods. Griffin, London 3rd ed.

Klahr, D. (1969), "Decision making in a complex environment: the use of similarity judgments to predict preferences," Management Science **15**, 595-618.

Krantz, D. H., R. D. Luce, P. Suppes, and A. Tversky (1971), Foundations of Measurement: Vol. 1, Additive and Polynomial Representations, Academic Press, London.

Lingoes, J., and E. E. Roskam (1973), "A mathematical and empirical study of two multidimensional scaling algorithms." Psychometrika **38**, monog. suppl., 1-93.

Luce, R. W., and H. Raiffa (1957), Games and Decisions, Wiley, New York.

Roskam, E. E. (1969), "Data theory and algorithms for nonmetric scaling," Nijmegen, Department of Psychology, mimeo.

Ryder, N. B., and C. F. Westoff (1965), Reproduction in the United States.

Shepard, R. N. (1964), "Attention and the metric structure of the stimulus space," Journal of Mathematical Psychology **1**, 54-87.

Steinheiser, F. H. (1970), "Individual preference scales within a multidimensional "similarities" space," Journal of Experimental Psychology **86**, 325-327.

Torgerson, W. S. (1958), Theory and Methods of Scaling, Wiley, New York.

Tucker, L. R., and S. J. Messick (1963), "Individual difference model for multidimensional scaling," Psychometrika **28**, 333-367.

Westoff, C. F., R. G. Potter, P. C. Sagi, and E. G. Mishler (1961), Family Growth in Metropolitan America, University Press, Princeton, N.J.